let $\rho_{matter} = 0 \quad p_{all\ but\ \Lambda} = 0$

then $$\dot{R}^2 + kc^2 = \frac{8\pi G}{3} \frac{c^2 \Lambda}{8\pi G} R^2$$
$$\dot{R}^2 + kc^2 = \frac{c^2\Lambda}{3} R^2$$

and we see for $\dot{R}$, $R >$ some value, kc^2 is negligible

$\Rightarrow$ also set $k = 0$

and $$\dot{R} = c\left(\frac{\Lambda}{3}\right)^{1/2} R$$

or $$R = R_i\, e^{c\left(\frac{\Lambda}{3}\right)^{1/2} t}$$

and R grows expotentially !

check units if $\frac{\Lambda c^2}{G}$ has units of g/cm^3

and G has units of $g^{-2}\,cm^2 \times g\text{-}cm/sec^2$
$= g^{-1}\,cm^3/sec^2$

$$\Rightarrow \quad \frac{\Lambda \times \frac{cm^2}{sec^2}}{g^{-1}\,cm^3/sec^2} = \frac{g\ \Lambda}{cm} = g/cm^3$$

$$\Rightarrow \Lambda = \frac{1}{cm^2} \quad ✓$$

before moving on to inflation:

take $\frac{\rho_m}{\rho_c} = 0.1$

take $H_0 = 50$

take T_0 = temp today = 2.73 K

and derive $1 + z_{eq} \approx \# \Omega_0 h^2$

where $\Omega(0) = \frac{\rho_m(0)}{\rho_c(0)} \equiv \Omega_0$

$$c^2 \rho_m(0)(1 + z_{eq})^3 = \rho_{rad}(0)(1+z)^4$$

$$\rho_m = \Omega_0 \rho_c = \Omega_0 \frac{3 H_0^2}{8\pi G}$$

$$H_0 \equiv (100 \text{ km/sec} - \text{mpc})\, h^2$$

$$1 + z_{eq} = \frac{c^2 \rho_m}{\rho_{rad}} = \frac{\Omega_0 \times 3 \times (3.33 \times 10^{-18})^2 h^2 c^2}{8\pi G \times 7.564 \times 10^{-15}\, T^4}$$

$T^4 = 55.56$

CGS units

$4.3 \times 10^4\, \Omega_0 h^2$ for ignoring neutrinos

if we put neutrinos in then we see that

$$\frac{\rho_r + \rho_{neu}}{\rho_c} = \Omega_r h^2 \approx 4.2 \times 10^{-5} \quad \text{(Peacock)}$$

or $\rho_r + \rho_{neu} \approx 4.2 \times 10^{-5} * \frac{3 H_0^2}{8\pi G} c^2 f$

$H_0^2 = 1\,(3.33 \times 10^{-18})^2$

ρ_r 5×10^{-30}

so

$$4.2 \times 10^{-5} = \Omega_r h^2$$

$$h^2 \Omega_r = \frac{\rho_{rad} + \rho_{nu}}{\rho_c}$$

$$\rho_c * 4.5 \times 10^{-5} = h^2 \Omega_r * \rho_c c^2 = \rho_{ra} + \rho_{nu}$$

$$\rho_{rad} = 7.56 \times 10^{-15} \times 55$$

$$\text{so } 3 \frac{(3.33 \times 10^{-18})^2 \times 9 \times 10^{20}}{8 \pi G} * 4.5 \times 10^{-5}$$

$= 2.67 \times 10^{-13} \times 3$

so 4.21×10^{-13}

1.9 ⇒ take my 4.2
and divide by 1.9
= 2.2

pretty close to 2.39 (more later)

entropy and the "big chrunch"

If you have photons in an expanding box with no heat input, then you can imagine the box is filled up with a photon "gas." Let's assume a black body distribution then as the box (or universe) expands, the # of photons remains constant because we have energy density goes as R^{-4} and # of $\frac{\text{photons}}{cm^3}$ $= \frac{\text{energy}}{cm^3\ h\nu} \propto \frac{\text{energy}}{cm^3\ T}$ and

= energy density $\propto T^4$

now entropy: specific entropy for the Universe is define as # photons / # of baryons and if both go as R^{-3}, we see the ratio stays constant and $\eta \equiv$ 1/(Specific Entropy is small) or the Entropy is high today ⇒ we infer the universe formed "hot"

=

now if $k = +1$, can Universe bounce? Guth and Petrosian disagree.

An executive summary is:

if you define entropy (or disorder) in the ordinary way as # of photons / baryon the universe won't bounce. But as Petrosian says, at high densities this definition might not hold, the universe might be far from thermodynamic equilibrium, and hence a bounce is possible side trip on G R - how well has it been tested etc.

- separate side trip on Mach's Principle

- Inertia (being able to feel acceleration is due to an interaction with the average mass of the universe

Newton said, there is an absolute coordinate system which defines all inertial frames [regardless of matter]

If Mach were correct - we should be able to see some subtle effects related to where we are in the Galaxy - The issue is not closed, for sure, but see below

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Einstein: Principle of Equivalence
"at every space-time in an arbitrary gravitational field it is possible to choose a locally inertial coordinate system such that, within a sufficiently small region of the point in question, the laws of nature take the same form in unaccelerated Cartesian coordinate systems in the absence of gravitation" [page 68 Weinberg]
i.e. any experiment in a freely falling elevator [system] anywhere in the universe will produce the same results - not quite Newton or Mach

Recent experiments trying to detect subtle effects due to the Milky Way Galaxy have been negative, favoring Einstein over Mach

Eötvös Experiment

- showed that inertial mass (measured by centrifugal motion of Earth) is the same as the gravitational mass for a wide range of substances (wood, glass, iron, copper) - so what? Well, different materials have different mixes of neutrons, protons, electrons, chemical bonds, etc. About 12 years ago (circ 1987) there were claims that Eötvös was wrong and a re analysis showed a definite dependence with neutron ratio ⇒ implied a new force! ⇒ but the idea was not confirmed, i.e. Eötvös was right after all.

How well is G R tested

(a) defection of light (but just need $E = mc^2$) - see homework however
(b) advance of perihelion of Mercury (43"/century) this is a good 1st order proof and has, by the way, imbedded in the calculat. that gravity waves or information about gravity travels as the speed of light in a vacuum. Note also the advance was known in 1895 but explanation in GR didn't come until 1916.

(c) the existence of gravitational radiation
- binary pulsar system
[also shows advance of perihelion]
(d) the existence of black holes
How good are these tests?
(a) as noted above, $E = mc^2$ and Newton alone gets within a factor of 2 - not a good test
[see homework]
(b) advance of perihelion might be due to other effects, but not likely, -
(c) gravitational radiation - difficult to predict from theory, so a little uncertain, but in general it seems pretty good - the only slight doubt is that maybe something else is causing the regular decay of the pulsar orbit. Check with LIGO/LISA
(d) there is still some nagging uncertainty about the existence of back holes, but again - seems pretty good (cf Scientific American, May '99)
$\Rightarrow$ bottom line is G R looks very good and hence general model of universe looks pretty good.

more on Black holes

$$R_H = \frac{2GM}{c^2} \approx 3\,\text{km}\,\frac{M_H}{M_\odot}$$

and the "density" = $\frac{M}{\frac{4}{3}\pi R^3}$ =

$$\frac{Mc^6}{\frac{4}{3}\pi 8 GM^3} = \frac{c^6}{\frac{4}{3}\pi 8 G^3 M^2} = \boxed{\frac{3c^6 G^{-3} M^{-2}}{32\pi}}$$

we'll find below that if we start the universe off at this critical density, for a given time, which implies a radius the universe will form hot and will expand; but locally, we could have density enhancements over a wide range of masses ⇒ primordial black holes – more later.

Also remember that GR or Newtonian gravity are mathematical descriptions of nature and hence they have no intrinsic "thruth"! Thus if they predict things that we know don't happen – we ignore the theory at this point.

Cosmology + Elementary Particle Physics

① Remember the motivation is strong for elemetary particle to do something "useful" or exciting such as putting the origin of the Universe on a mathematic footing ⇒ even if it doesn't work [yet], people are not giving up.

② Currently, the models are not in a finished state, so why talk about it? Because so much is being said / written

Overview: Details to follow –

Ⓐ by involving new laws of physics it is possible to produce a de Sitter type cosmology in the early Universe ⇒ Inflation. The basic problem is that we can't really seem to get it to work

Ⓑ can use astrophysics to set limits on the existence of "bizzare particles" from relatively tame quarks to exotic stuff

Ⓒ We can (and have to some extent) address entropy and the arrow of time

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Particles and Field Theory

Field Theory – says there is no interaction [force] at a distance [e.g. gravity, electrical] takes place without the exchange of particles (the opposite of GR)

⇒ The discussion of forces inevitably leads to a discussion of types of particles.

⇒ Standard model for elementary particles

QCD + quarks + gluons [strong force]

Electroweak – leptons [electrons, neutrinos W, Z bosons etc]

only reason we need to know this is to have a concept of symmetry + symmetry breaking

Standard model describes 3 forces that are major players in particle accelerator physics

- strong - holds nuclei together
- weak - causes β decay
- electric - coulomb's law, etc.

electro weak said go to high enough energies and electric + weak become indistinguishable